TECHNICAL NOTE

Natural convection in a vertical slot filled with porous medium

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Natural convection in a porous rectangular cavity with isothermal vertical walls at different temperatures has been studied. It is found that for a given Rayleigh number Ra, the convection Nusselt number Nu_c (or Nu-1), which represents the enhancement of heat transfer by convection, is inversely proportional to the height/width aspect ratio H as the latter increases. This leads to a very simple correlation expressed as $Nu_c H = a(H > H_m)$, where constant a and marginal aspect ratio H_m are determined numerically. It is demonstrated that this new correlation is consistent with the observation that for a given Ra, the effect of the top and bottom adiabatic ends is limited to a fixed length; beyond that, an asymptotical parallel flow, which coincides with the analytical solution for an infinitely long enclosure, prevails around the mid-height of the enclosure. It is suggested that instead of Nu, Nu_c (= Nu-1) be used in the correlation of Nu, Ra and H in order to cover results for high H and low Ra.

Keywords: natural convection; porous medium; vertical slot; heat-transfer correlation

Introduction

Natural convection in a vertical porous slot has been extensively studied. However, only a few in the literature have discussed the case of large height-to-width ratio H. Burns et al.[1] developed an analytical Nu–Ra correlation in which one coefficient was obtained by comparing and matching with numerical results. Walker and Homsy[2] discussed shallow cavities, i.e., $H \to 0$, and the case of $Ra \to \infty$ with H fixed. For other media and boundary conditions, Haajizadeh et al.[3] and Vasseur et al.[4] also discussed the effect of the appearance of parallel flows for large Hs.

The present paper provides a clear physical picture as well as a simple correlation expression of the heat transfer behavior as H is increased toward asymptotically high values ($H \to \infty$). It is found that for a given Rayleigh number Ra, the convection Nusselt number Nu_c (or Nu-1), which represents the enhancement of heat transfer by convection, is inversely proportional to H. In equation form, $Nu_c H = a$ or $Nu = a(Ra)/H + 1$ ($H > H_m$), where a and marginal aspect ratio H_m vary with Ra. The quantity a is obtained from numerical values for Nu_c and H_m, i.e., $a = (Nu_m - 1)H_m$, and the marginal aspect ratio H_m is numerically determined by gradually increasing H until parallel flow pattern appears. Numerical calculations for $H > H_m$ are also carried out in order to verify the validity of the analytical expression.

Solution procedure

The numerical scheme is briefly described as follows. The vertical walls of the rectangular enclosure are isothermal but at different dimensionless temperatures, $\theta = 1$ and $\theta = 0$, respectively. The top and bottom ends are insulated. Numerical solutions are obtained by solving the dimensionless Darcy-Boussinesq equations given by the following:

$$\nabla^2\psi = Ra \frac{\partial \theta}{\partial x} \tag{1}$$

$$\nabla^2\theta = u\frac{\partial \theta}{\partial x} + v\frac{\partial \theta}{\partial y} \tag{2}$$

In these equations, u and v are the dimensionless horizontal and vertical velocities; ψ is the stream function; θ is the dimensionless temperature; Ra is the Darcy-Rayleigh number based on the enclosure width. All these symbols have their traditional definitions.[1–4]

Equations 1 and 2 are discretized into central-difference algebraic equations that are then solved iteratively with the SOR method. The scheme is justified by the good agreement between the results it generates and the data in the literature, and also by the fact that doubling the number of grid points or increasing the convergence criteria by one order will not change the average Nusselt numbers obtained by more than 1 percent.

Results and discussion

It is known that for a given Ra, the effect of the top and bottom adiabatic ends is limited to a fixed length—beyond that, the parallel flow pattern with the pure-conduction distribution of isotherms appears around the midheight, which is the straightforward solution of Equations 1 and 2 for infinite H[1]:

$$\psi = \frac{Ra}{2}(x - x^2) \tag{3}$$

$$u = 0, \qquad v = \frac{Ra}{2}(1 - 2x) \tag{4}$$

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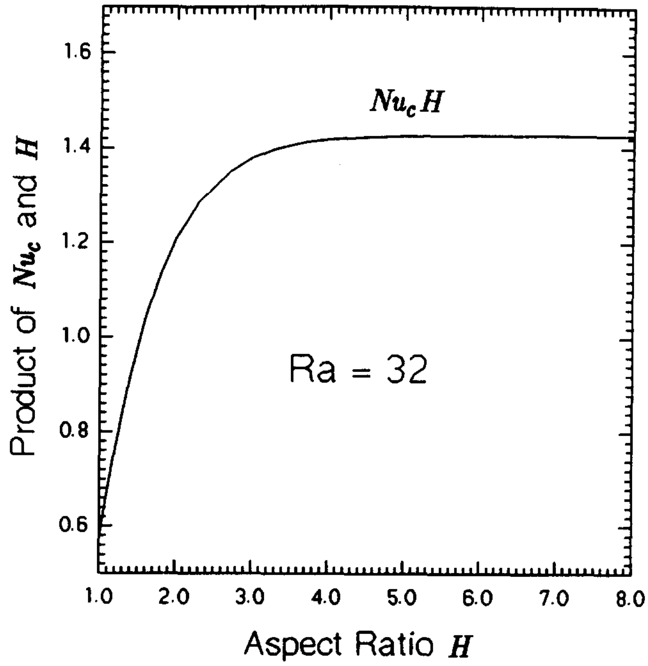

Figure 1 Variation of $Nu_c H$ with H for Ra = 32

$$\theta = 1 - x \qquad (5)$$

Extensive calculations are carried out to confirm that as H increases gradually from the value of 1.0, the velocity and stream-function profiles at midheight $y = 0.5H$ monotonically as well as asymptotically approach the above analytical solution, and are practically identical for values of H larger than a fixed value H_m. The length of the parallel flow section is consequently $H - H_m$. In the following, it will be shown that this leads to a simple correlation of Nu and H for $H > H_m$.

Consider two enclosures with different aspect ratios H_1 and H_2 ($H_1 > H_m$; $H_2 > H_m$), and Nusselt numbers Nu_1 and Nu_2, respectively. We divide the Enclosure 1 (H_1) into two portions, H_m (including two end regions) and $H_1 - H_m$ (around the middle height), with H_m representing nonparallel flows ($Nu = Nu_m$) and $H_1 - H_m$ parallel flows ($Nu = 1$). The average Nusselt number for the Enclosure 1 (H_1) is thus

$$Nu_1 = \frac{(H_1 - H_m) \times 1 + H_m \times Nu_m}{H_1} \qquad (6)$$

Replacing H_1 in Equation 6 with H_2 yields Nu_2 for the Enclosure 2 (H_2) in a similar form. By eliminating H_m and Nu_m we have

$$\frac{Nu_2 - 1}{Nu_1 - 1} = \frac{H_1}{H_2} \qquad (7)$$

or

$$Nu_c H = a \quad \text{or} \quad Nu = \frac{a(Ra)}{H} + 1 \qquad (H \geqslant H_m) \qquad (8)$$

where a and H_m are constants for a fixed Ra. This can also be regarded as the following asymptotical process:

$$\lim_{H \to \infty} Nu_c H = a \qquad (9)$$

It is interesting to find that when H approaches infinity, Nu_c and $1/H$ are infinitesimals of the same order so that the value of the product $Nu_c H$ approaches a non-zero constant for a non-zero Ra. Figure 1 shows the variation of $Nu_c H$ with H for a given Ra. The values are very close to an asymptotical constant for Hs greater than about 4.0. This suggests that the results for $H = 4.0$ (H_m) can be used to obtain the constant a as follows:

$$a = (Nu_m - 1)H_m \qquad (10)$$

In this approximation, H_m is chosen as the aspect ratio at which ψ_{max} differs by 3 percent from the limiting value for infinite H. For Ra = 32, this criterion generates $H_m = 3.94$ by increasing H from a lower value. In Figure 2, the Nusselt numbers from the analytical correlation are compared with those from

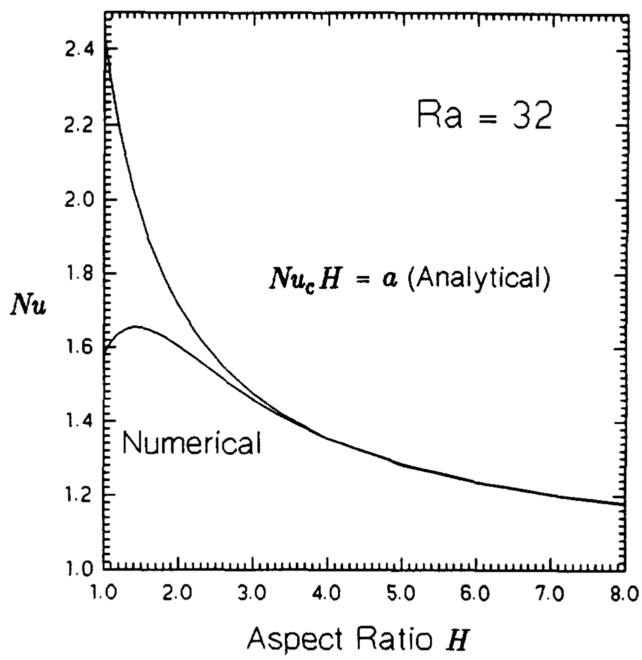

Figure 2 Comparison of results obtained from the relation $Nu_c H = a$ with numerical ones; Ra = 32

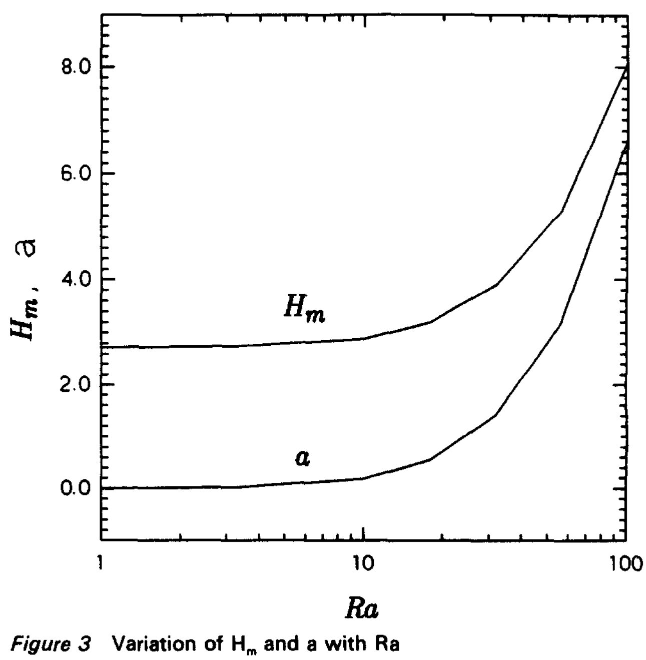

Figure 3 Variation of H_m and a with Ra

numerical results for Ra = 32. The agreement is very satisfactory. For Hs greater than about 3.5 the two curves almost overlap with each other. The validity and efficiency of the 3 percent criterion is therefore justified. It is important to note that the analytical expression is not correlated from the numerical results for $H > H_m$ $(=3.94)$. The numerical calculations for the region $H > H_m$ are carried out only to obtain data in order to compare with the analytical results.

Marginal aspect ratios determined with the criterion above are shown in Figure 3. The H-Ra plane is divided into two areas: convection without parallel flows ($H \leqslant H_m$) in which numerical simulation is the only way to obtain results, and convection with parallel flows ($H > H_m$) in which the analytical correlation is valid. H_m is found to be a monotonic function of Ra. At small Ra, it approaches a fixed value of about 2.7, which could be considered the minimum aspect ratio for an enclosure to accommodate parallel flows. Also given in Figure 3 are the values of quantity a for the analytical correlation. It indicates the intensity of heat transfer that reflects the strength of the convective flow. It increases monotonically with increasing Ra and approaches zero when Ra decreases to very small values.

Acknowledgment

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